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Original Research Article

USING THE GRAY VERHULST MODEL TO EXPLORE THE DEVELOPMENT OF THE ELDERLY POPULATION AND ELDERLY FAMILIES IN NANTOU COUNTY, TAIWAN

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Abstract

Purpose of the Study: The Verhulst model is a biological growth model proposed by the German biomathematician Verhulst in 1837. Population growth will also be limited by environmental and other development factors and tend to be stable. Its characteristic is that the trend predicted by the model will tend to a fixed value and reach stability. The gray Verhulst model combines the characteristics of the gray prediction GM(1,1) model and the Verhulst model, and adds restrictive development factors to the gray prediction model to infer the possible stable growth and development of the population.

Methodology: The research method of this study is the gray Verhulst model, which combines the characteristics of the gray prediction GM(1,1) model and the Verhulst model.

Main Findings: Gray prediction is a prediction model that has been widely promoted in recent years because it often requires only a small number of samples to obtain high prediction accuracy (more than 90%). This study is based on the population data of Nantou County, Taiwan from 2003 to 2020, and uses the Gray Fairhast model to estimate the number of elderly people and the growth of elderly families in Nantou County, Taiwan. The results show that the number of elderly households in Nantou County, Taiwan is expected to increase by 313 households in 2023, bringing the total number of elderly households to 14,853. The elderly population is expected to increase by 1,881 people, bringing the total number of elderly people to 93,424.

Applications of this study: This project provides a unique perspective, based on the biological growth model proposed by German biomathematician Verhulst in 1837. The trend predicted by the model will tend to a fixed value and reach stable characteristics, and population growth will also be affected by environmental and other developments. factors to predict future population development.

Novelty of this Study: The new contribution offered here is a reference for government departments to propose elderly population policy and investment in public facilities construction.

Keywords: Gray Verhulst model, Population Growth, Elderly Population, Elderly Household.

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Introduction

According to the definition of the United Nations World Health Organization (WHO), people over 65 years old are the elderly. When the elderly population accounts for more than 7% of the total population, the society is called an "aging/aging society"; when it reaches 14%, the society is called an "aged society". society). When the elderly population accounts for more than 20% of the total population, the society is called a "super-aged society" (Chen Meiru, 2010; Chen Junhe, 2021). According to statistics from the Ministry of Interior, at the end of January 2020, China's elderly population (over 65 years old) was 3.804 million, accounting for 16.2% of the total population since it exceeded the threshold of 14.0% of the "aging society" defined by the United Nations at the end of March 2017. Compared with the end of 2020, it increased by 17,000 people or 0.4%. Compared with the end of 2014 (2.939 million people), it increased by 865,000 people or 29.4% (Executive Yuan Accounting Office, 2021). The discussion and understanding of population growth and changes in the number of households is one of the important basis for the government to formulate population and public construction policies (Chun-Ho Chen, 2019; Chen Junhe, 2021, 2012, 2010, 2008, 2007a, 2007b).

Nantou County, Taiwan, is located in the center of Taiwan Island, covering an area of 4,106 square kilometers. It is the second largest county in Taiwan Province and the only inland county in the province that is not close to the sea. The administrative region is divided into 1 city, 4 towns and 8 townships, namely Nantou City, Puli Town, Caotun Town, Zhushan Town, Jiji Town, Mingjian Township, Zhongliao Township, Lugu Township, Shuili Township, Yuchi Township, and Guoxing Township. , Xinyi Township, Renai Township and other 13 towns and cities. Among them, Xinyi and Renai are mountainous townships; Yuchi Township is flatland aboriginal township, as shown in Figure 1 (Nantou County Council, Taiwan, 2022). The elderly population ratio (over 65 years old) in Nantou County, Taiwan, from 2010 to 2021 is summarized in Table 1. From Table 1, we can see that the elderly population ratio in Nantou County, Taiwan, in 2013 reached 14.3%, indicating that it has entered an elderly society. In 2020 In 2016, the proportion of the elderly population (over 65 years old) was as high as 18.65%. This article subsequently uses the Gray Fairhast model to estimate the development trend of the number of elderly households and the number of elderly people in Nantou County, Taiwan in 2023.

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Ratio	13.74	13.94	14.30	14.71	15.21	15.86	16.52	17.17	17.86	18.65

Table 1. Ratio of elderly population (over 65 years old) in Nantou County, Taiwan, 2011-2021
Data source: Statistical Information Network of the Republic of China, 2022.

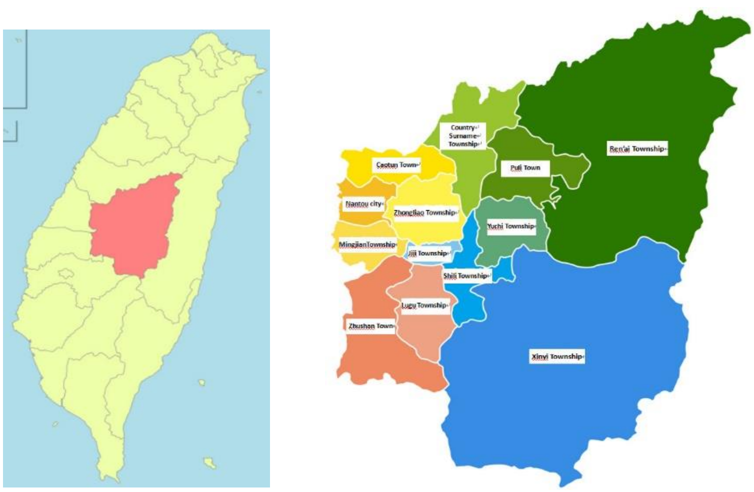

Figure 1. Administrative area map of Nantou County, Taiwan
Source: Wikipedia (https://zh.wikipedia.org/zh-tw/%E5%8D%97%E6%8A%95%E7%B8%A3)

Research Methods

(A) Gray prediction mode

Traditional long-term regular prediction models often require the collection of a large number of data samples, which is not economical. However, only a small sample can be used to obtain a gray prediction model with high prediction accuracy (90%), which has gradually been widely used (Deng Julong, 2002). Gray prediction is one of the contents of gray system theory. It is based on the GM(1,1) model to find out the dynamic state and future development of each element in a series, thereby predicting the future state of the system. The main advantage is that it does not require too much data (4 transactions), which is different from traditional regression analysis. Its characteristics are as follows (Wu Hanxiong et al., 1996):

1. Gray prediction requires less data. Just select an appropriate amount of data based on the actual situation. Even just four pieces of data can be used to model and predict, and accurate results can be obtained.
2. Although GM is based on a deeper foundation of advanced mathematics, its calculation steps are not cumbersome.
3. Under normal circumstances, gray prediction does not require too many related factors, so it can simplify the data collection work.
4. Gray forecasting can be used for both short-term and medium- and long-term forecasting.
5. Gray prediction has high accuracy. With the same small number of samples, the prediction error of the model is smaller than that of other methods.

The gray prediction model is designed to explore systems with incomplete information. It can make full use of limited data information to conduct analysis and research, and has good prediction capabilities. The following explains the modeling method of gray prediction model GM(1,1) (Deng Julong, 1988). The first-order differential equation of the GM(1,1) mode is:

$$\frac{dx^{(1)}}{dt} + \alpha x^{(1)} = b \quad \textbf{(1)}$$

Among them, t is the independent variable of the system, α is the development coefficient, b is the gray control variable, α and b are the undetermined parameter of the model. Suppose the original sequence looks like this:

$$x^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\right) \quad \textbf{(2)}$$

When modeling a gray system, it is necessary to first accumulate the original sequence to provide intermediate information for modeling and weaken the randomness of the original sequence. Also define $x^{(1)}$ as the linear AGO sequence of $x^{(0)}$, that is

$$\begin{aligned} x^{(1)} &= \left(x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\right) \\ &= \left(\sum_{k=1}^{1} x^{(0)}(k), \sum_{k=1}^{2} x^{(0)}(k), \ldots, \sum_{k=1}^{n} x^{(0)}(k)\right) \end{aligned} \quad \textbf{(3)}$$

From equations (1) and (3) and the least squares method, the coefficient $\hat{\alpha}$ is obtained:

$$\hat{\alpha} = \begin{bmatrix} \alpha \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y \quad \textbf{(4)}$$

where the accumulation matrix B is:

$$B = \begin{bmatrix} -\frac{1}{2}\left[x^{(1)}(1) + x^{(1)}(2)\right] & 1 \\ -\frac{1}{2}\left[x^{(1)}(2) + x^{(1)}(3)\right] & 1 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ -\frac{1}{2}\left[x^{(1)}(n-1) + x^{(1)}(n)\right] & 1 \end{bmatrix}$$

The constant term vector Y is:

$$Y = \left[x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\right]^T$$

Substituting the calculated coefficient $\hat{\alpha}$ into the differential equation, and solving equation (1), the approximate relationship can be obtained as:

$$\hat{x}^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{\alpha}\right]e^{-\alpha k} + \frac{b}{\alpha} \tag{5}$$

Among them $x^{(1)}(1) = x^{(0)}(1)$, by performing an inversed-accumulated generating operation (IAGO) on the sequence obtained by equation (5), the sequence to be restored can be obtained, as shown in equation (6):

$$\hat{x}^{(0)}(k) = \left[x^{(0)}(1) - \frac{b}{\alpha}\right]e^{-\alpha(k-1)}(1 - e^{\alpha}) \tag{6}$$

Let k=1,2,…,n, and the restored sequence is:

$$\hat{x}^{(0)} = \left(\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \ldots, \hat{x}^{(0)}(n)\right)$$

After the above generation process and model construction, it is necessary to conduct an accuracy test to understand the error e between the predicted value and the actual value. The residual test method is to compare the residual between the actual value and the predicted value. The formula is as follows (Deng Julong, 1988):

$$e(k) = \left|\frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}\right| \text{ x } 100\%, \quad k=2,3,\ldots,n \tag{7}$$

The accuracy is $1 - e(k)$, and if the average accuracy is greater than 90%, the prediction performance of this model is good.

(B) Gray Verhulst model

The Verhulst model is a biological growth model proposed by the German biomathematician Verhulst in 1837. Its characteristic is that the prediction trend of the model will tend to a certain fixed value and reach saturation stability. This prediction model is a nonlinear prediction model; Gray Verhulst is This feature is combined with the gray prediction model. This model not only retains the advantages of the original gray prediction model, but also corrects the limitation that the gray prediction model is only suitable for predicting linear data, improving the prediction accuracy and application of the gray prediction model for nonlinear data types. Scope (Jiang Boting, 2005).

The gray Fairhast model is based on the characteristics of the GM(1,1) model, and a development-limiting term is added to the GM(1,1) model to meet the actual saturation situation. Its basic mathematical model is as follows (8) (Wen Kunli et al., 2006):

$$\frac{dx^{(1)}}{dx}+ax^{(1)}=b(x^{(1)})^2 \qquad \textbf{(8)}$$

Expressed in the form of gray theory difference equation, it becomes

$$x^{(0)}(k)+az^{(1)}(k)=b(z^{(1)}(k))^2 \Rightarrow x^{(0)}(k)=-az^{(1)}(k)+b(z^{(1)}(k))^2 \qquad (9)$$

1. Just like the solution to the GM(1,1) model, by substituting all the data into equation (9), we can get

$$x^{(0)}(2)=-az^{(1)}(2)+b(z^{(1)}(2))^2$$

$$x^{(0)}(3)=-az^{(1)}(3)+b(z^{(1)}(3))^2$$

$$x^{(0)}(4)=-az^{(1)}(4)+b(z^{(1)}(4))^2$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$x^{(0)}(n)=-az^{(1)}(n)+b(z^{(1)}(n))^2 \qquad \textbf{(10)}$$

2. Simplify equation (10), use the matrix $\hat{a}=(B^TB)^{-1}B^TY$, where Y= B$\hat{a}$, to find the value of a · b .

$$Y=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ x^{(0)}(4) \\ \cdots\cdots \\ x^{(0)}(n) \end{bmatrix} \qquad B=\begin{bmatrix} -z^{(1)}(2) & (z^{(1)}(2))^2 \\ -z^{(1)}(3) & (z^{(1)}(3))^2 \\ -z^{(1)}(4) & (z^{(1)}(4))^2 \\ \cdots\cdots & \\ -z^{(1)}(n) & (z^{(1)}(5))^2 \end{bmatrix} \qquad \hat{a}=\begin{bmatrix} a \\ b \end{bmatrix} \qquad \textbf{(11)}$$

3. Use the parameter method to find the values of a and b .

$$a=\frac{DH-GE}{FG-D^2} \qquad b=\frac{EF-DE}{FG-D^2} \qquad (12)$$

among them： $D=\sum_{k=2}^{n}(z^{(1)}(k))^3$ · $E=\sum_{k=2}^{n}z^{(1)}(k)\times x^{(0)}(k)$ · $F=\sum_{k=2}^{n}(z^{(1)}(k))^2$

$$G=\sum_{k=2}^{n}(z^{(1)}(k))^4 \cdot H=\sum_{k=2}^{n}(z^{(1)}(k))^2\times x^{(0)}(k)$$

4. Substitute the obtained values of a and b into the solution of Verhulst's quasi-differential equation. The predicted value is:

$$\hat{x}^{(1)}(k)=\frac{\frac{a}{b}}{1+(\frac{a}{b}\times\frac{1}{x^{(0)}(1)}-1)e^{a(k-1)}} \qquad k\geq 2 \qquad \textbf{(13)}$$

5. Find out from the definition of AGO

$$\overset{\Lambda}{x}^{(0)}(k)=\overset{\Lambda}{x}^{(1)}(k)-\overset{\Lambda}{x}^{(1)}(k-1).$$

$$\overset{\Lambda}{x}^{(0)}(k)=\frac{(1-e^{-a})\times\frac{a}{b}\times[1+(\frac{a}{b}\times\frac{1}{x^{(0)}(1)}-1)e^{a(k-1)}]}{[1+(\frac{a}{b}\times\frac{1}{x^{(0)}(1)}-1)e^{a(k-1)}][1+(\frac{a}{b}\times\frac{1}{x^{(0)}(1)}-1)e^{a(k-2)}]}\quad k\geq 2 \qquad (14)$$

From equation (13), we know that if a <0, then $\lim_{k\to\infty}\overset{\Lambda}{x}^{(1)}(k)\to\frac{a}{b}$, that is, the saturation point of equation (13) is $\frac{a}{b}$. This value is the limit value caused by limiting the development direction, that is, the saturation point of the predicted value $\overset{\Lambda}{x}^{(0)}(k)$.

Results and Discussion

(A) Estimated number of elderly households in Nantou County, Taiwan

The number of elderly households in Nantou County, Taiwan, in the past 12 years (2009-2021) is shown in Table 2. The gray Fairhast model was used to analyze the difference in the number of elderly households every two years. Table 3 is used to estimate the number of elderly households in Nantou County, Taiwan in 2022.

(2009-2021) (household/year)

Year	2004	2005	2006	2007	2008	2009	2010	2011	2012
Number of households	--- (lack)	--- (lack)	--- (lack)	--- (lack)	--- (lack)	9,085	8,820	9,364	9,642

Year	2013	2014	2015	2016	2017	2018	2019	2020	2021
Number of households	10,082	10,430	10,992	11,668	12,434	13,023	12,744	13,607	14,540

Table 2. Number of elderly households in Nantou County, Taiwan

Source: Ministry of Interior Real Estate Information Platform, 2021.

Year	2011	2013	2015	2017	2019	2021	2023
Number of households	9,364	10,082	10,992	12,434	12,744	14,540	?
Difference in number of households in 3 years	718	910	1,442	310	1,796	?	

Table 3. Extracted numerical difference series of the number of elderly households in Nantou County, Taiwan (household/year)

Source: this study.

Step 1: Build the basic sequence： $x^{(0)}$ =(718, 910, 1442, 310, 1796)

Step 2: Create AGO sequence： $x^{(1)}$ =(718, 1628, 3070, 3380, 5176)

Step 3: Create a mean generating sequence： $z^{(1)}$ =(-----, 1173, 2349, 3225, 4278)

Step 4: According to equation (12), use the parameter method to find the value of a and b.

$$D=\sum_{k=2}^{n}(z^{(1)}(k))^{3}=1.2641\times 10^{11}$$

$$E=\sum_{k=2}^{n}z^{(1)}(k)\times x^{(0)}(k)=1.3137726\times 10^{7}$$

$$F=\sum_{k=2}^{n}(z^{(1)}(k))^2 = 3.5595639 \times 10^7$$

$$G=\sum_{k=2}^{n}(z^{(1)}(k))^4 = 4.75449 \times 10^{14}$$

$$H=\sum_{k=2}^{n}(z^{(1)}(k))^2 \times x^{(0)}(k) = 4.5302064246 \times 10^{10}$$

$$a=\frac{DH-GE}{FG-D^2} = -0.550283589,\quad b=\frac{EF-DE}{FG-D^2} = -0.001758046$$

Step 5: Substitute the required values of a and b into the solution of Verhulst's quasi-differential equation, and obtain the prediction model as:

$$\hat{x}^{(1)}(k)=\frac{\frac{a}{b}}{1+(\frac{a}{b}\times\frac{1}{x^{(0)}(1)}-1)e^{a(k-1)}}$$

Step 6: Using $\lim_{k\to\infty}\hat{x}^{(1)}(k)\to\frac{a}{b}$, the limit point of the predicted value $\hat{x}^{(0)}(k)$ is obtained as:

$$\frac{a}{b}=\frac{DH-GE}{EF-DE}=\frac{-0.550283589}{-0.001758046}\doteqdot 313$$

According to the calculation results, the limit point of change in the number of elderly households in Nantou County, Taiwan can be predicted. The change value from 2021 to 2023 is an increase of 313 households. That is, the limit value of the number of elderly households in 2023 is: 14,540 + 313= 14,853 households.

(A) Estimation of the elderly population in Nantou County, Taiwan

The number of elderly people in Nantou County, Taiwan in the past 18 years (2003-2020) is shown in Table 4. The gray Fairhast model was used to analyze the difference in the number of elderly people every three years. The obtained values are summarized as follows: Table 5 shows the estimate of the elderly population in Nantou County, Taiwan in 2023.

Year	2003	2004	2005	2006	2007	2008	2009	2010	2011
Population	62,460	64,421	66,234	67,989	69,325	70,294	71,073	71,366	71,845

Year	2012	2013	2014	2015	2016	2017	2018	2019	2020
Population	72,541	73,941	75,641	77,509	80,135	82,780	85,365	88,253	91,543

Table 4. Number of elderly people in Nantou County, Taiwan (2003-2020) (person/year)
Data source: Statistical Information Network of the Republic of China, 2022

Year	2005	2008	2011	2014	2017	2020	2023
Population	66,234	70,294	71,845	75,641	82,780	91,543	?
Difference in population in 3 years	4,060	1,551	3,796	7,139	8,763	?	

Table 5. Extracted difference series of elderly population in Nantou County, Taiwan (person/year)
Source: This study.

Step 1: Create a basic sequence: $x^{(0)}$ = (4060, 1551, 3796, 7139, 8763)

Step 2: Create AGO sequence: $x^{(1)}$ = (4060, 5611, 9407, 16546, 25309)

Step 3: Establish a mean generation sequence: $z^{(1)}$ = (-----, 4835.5, 7509, 12976.5, 20927.5)

Step 4: According to equation (12), use the parameter method to find the value of a and b.

$$D=\sum_{k=2}^{n}(z^{(1)}(k))^{3} = 1.1887 \times 10^{13}$$

$$E=\sum_{k=2}^{n}z^{(1)}(k)\times x^{(0)}(k) = 3.12030940 \times 10^{8}$$

$$F=\sum_{k=2}^{n}(z^{(1)}(k))^{2} = 6.86116950 \times 10^{8}$$

$$G=\sum_{k=2}^{n}(z^{(1)}(k))^{4} = 2.2389 \times 10^{17}$$

$$H=\sum_{k=2}^{n}(z^{(1)}(k))^{2}\times x^{(0)}(k) = 5.29028 \times 10^{12}$$

$$a=\frac{DH-GE}{FG-D^{2}} = -0.566417688, \quad b=\frac{EF-DE}{FG-D^{2}} = -0.000301179$$

Step 5: Substitute the required values of a and b into the solution of Verhulst's quasi-differential equation, and obtain the prediction model as:

$$\hat{x}^{(1)}(k)=\frac{\frac{a}{b}}{1+(\frac{a}{b}\times\frac{1}{x^{(0)}(1)}-1)e^{a(k-1)}}$$

Step 6: Using $\lim_{k\to\infty}\hat{x}^{(1)}(k)\to\frac{a}{b}$, the limit point of the predicted value $\hat{x}^{(0)}(k)$ is obtained as:

$$\frac{a}{b}=\frac{DH-GE}{EF-DE}=\frac{-0.566417688}{-0.000301179}\fallingdotseq 1,881$$

According to the calculation results, the limit point of the change value of the elderly population in Nantou County, Taiwan can be predicted. The change value from 2020 to 2023 is an increase of 1,881 people. That is, the limit value of the elderly population in 2023 is: 91,543 + 1,881 = 93,424 people.

Conclusion

Nantou County, Taiwan the elderly population ratio in Nantou County, Taiwan, reached 14.3% in 2013, indicating that it has entered an elderly society. In 2020, the elderly population ratio (over 65 years old) is as high as 18.65%. In addition, the gray Fairhast model is used to estimate the number of elderly households and the total elderly population in Nantou County, Taiwan. The number of elderly households in Nantou County, Taiwan is expected to increase by 313 households in 2023, and the total number of elderly households is 14,853. The population is estimated to increase by 1,881 people, and the total elderly population is 93,424. The results can provide a reference for government departments to propose Yilan County's elderly population policy and investment in public facilities construction.

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